**Relative $t$-designs on one shell of Johnson association schemes**

Yan Zhu
*University of Shanghai for Science and Technology, Shanghai, China*
yanzhu2017@gmail.com

The concepts of relative $t$-designs in Q-polynomial and P-polynomial association schemes were respectively introduced by Delsarte [2] in 1977 and Bannai-Bannai-Suda-Tanaka [1] in 2015.

In this talk, we will discuss relative $t$-designs on one shell in Johnson association scheme $J(v,k)$ for both P- and Q-polynomial structures. For any fixed point $u_0 \in \binom{V}{k}$, the $r$-th shell of $J(v,k)$ w.r.t. $u_0$ is defined by $X_r = \{x \in \binom{V}{k} : |x \cap u_0| = k - r\}$. Each nontrivial shell $X_r$ of $J(v,k)$ is known to be a commutative association scheme which is the product of two smaller Johnson association schemes, but it is not a Q-polynomial association scheme anymore. However, Martin [3] defined $\mathcal{T}$-designs in the product of Q-polynomial association schemes, where $\mathcal{T}$ is a subset of $\mathbb{N} \times \mathbb{N}$ satisfying some condition. We prove the following result.

Let $(Y, w)$ be a relative $t$-design in $J(v,k)$ w.r.t. $u_0$ on one shell $X_r$. Then $(Y, w)$ is a weighted $\mathcal{T}$-design in $X_r$ (as product association scheme), where

(i) $\mathcal{T} = \{(t_1, t_2) \mid 0 \le t_1 + t_2 \le t\}$ for Q-polynomial structure of $J(v,k)$.

(ii) $\mathcal{T} = \{(t_1, t_2) \mid 0 \le t_1, t_2 \le t\}$ for P-polynomial structure of $J(v,k)$.

We also study the existence problem of tight relative $t$-designs for $t = 2, 3$ for both P- and Q-polynomial structures. For P-polynomial structure, we propose an algorithm to construct tight relative 2-designs. All known examples of tight relative 3-designs for both P- and Q-polynomial structures are constructed from Hadamard 2-$(4u-1, 2u-1, u-1)$ designs.

This talk is based on joint works with Eiichi Bannai and Naoki Watamura.

# References

[1] E. Bannai, Et. Bannai, S. Suda, and H. Tanaka. On relative $t$-designs in polynomial association schemes. *Electronic J. Comb.*, 22(4):4–47, 2015.

[2] P. Delsarte. Pairs of vectors in the space of an association scheme, *Philips Res. Rep.*, 32(5–6):373–411, 1977.

[3] W. J. Martin, Designs in product association schemes, *Des. Codes Cryptogr.*, 16:271–289, 1999.